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#### RESEARCH ARTICLE

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## Cultural foundations of a mathematician's thinking: a Psychobiographical exploration of Zoltán Paul Dienes and his cognitive development

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#### **ABSTRACT**

This psychobiographical study utilises Lev Vygotsky's sociocultural theory to explore the impact of diverse cultural experiences on the intellectual growth of Zoltán Paul Dienes, a prominent mathematician and theorist of mathematical education. Vygotsky's sociocultural theory is particularly relevant for understanding the impact of culture on intellectual development. According to Vygotsky, cognitive development is shaped by social and cultural interactions, and the tools and practices of a given culture. The study employs a taxonomy of cognitive tools to categorise the influences young Dienes encountered in Hungary, Austria, France, United Kingdom and elsewhere. By examining the presence and frequency of these tools in his formative years, the research sheds light on the origins of Dienes's principles in mathematical education. The research enriches psychobiography by integrating sociocultural theory, providing valuable insights into the relationship between culture, cognitive tools, and intellectual growth in knowledgeable individuals. Ultimately, this study contributes to a better comprehension of the origins and principles behind Dienes's theory of mathematical education.

#### **ARTICLE HISTORY**

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#### **KEYWORDS**

Psychobiography; Vygotsky; Zoltán Paul dienes: cognitive tool; sociocultural theory

### Lev Vygotsky' theory on cognitive development

The concept of culture refers to the collective patterns of beliefs, values, customs, behaviours, language, art, rituals, and knowledge that are learned, transmitted, and shared by members of a community. Lev Vygotsky's sociocultural theory of cognitive development emphasises the crucial role of culture and social interaction in shaping an individual's cognitive development. According to Vygotsky, cognitive development is not solely an internal, individual process but rather a product of cultural and social influences. Cultures provide their members with diverse tools, including the ones called "cognitive tools" which serve as a means of extending cognitive abilities beyond the individual's current capacity, and allow individuals to internalise knowledge, create mental representations. Cognitive tools range from external physical tools (e.g. maps, diagrams, charts) to internal psychological ones (number systems, language, etc.). They trigger the development of "higher order mental functions", such as imaginative play, symbolic thinking, creativity,

abstract reasoning, plan and goal setting, metacognition, self-regulation, critical thinking, problem-solving and conceptual understanding. All in all, cognitive tools play a crucial role in the individual's cognitive development. Thus, Vygotsky's theory suggests that cognitive development is essentially a social and cultural process that occurs through interactions with more knowledgeable individuals and the use of cultural tools, such as language, symbols, and other artefacts (Vygotsky, 1962, 1978; Veer & Valsiner, 1991).

#### Psychobiographical explorations of Zoltán Paul dienes's childhood

In my study, I investigate how the cognitive tools provided by diverse cultures to Zoltán Paul Dienes in his childhood impacted his intellectual growth. Zoltán Dienes (1916–2014), the influential theorist of mathematical education, grew up in several different countries including Hungary, Austria, France and the United Kingdom. Apart from being somewhat immersed in the mainstream (macro) culture of these countries, I will point out that he was rather



influenced by the specific cultural tools provided by some smaller communities. Since the notion of culture encompasses both national/regional (macro) aspects, and the specific and shared cultures of smaller groups, subcultures, organisations, and even intimate social circles (micro), cognitive tools provided by both types should be investigated. Probably it is the micro aspects of culture that have the greatest impact on the development of the individual.

Psychobiographical research provides a valuable lens for understanding the complex interplay between culture and intellectual development. In my recent study (Bálint, 2021), I reviewed the diverse cultural contexts in which Zoltán Dienes was raised and summarised their contribution to his personality development. I pointed out that Zoltán Dienes's upbringing was marked by a constant shift between various cultural contexts, languages, and educational systems. Below is a concise overview of the key cultural settings and their effects throughout the course of his upbringing.

Home in Hungary: Zoltán was born into a highly educated Hungarian family. His parents were intellectuals who were connected to prominent members of the Hungarian intelligentsia. Zoltán's early experiences in their Budapest home were marked by Hungarian culture and the nurturing environment of his parents.

Montessori Children's Home, Vienna (Austria): At the age of 3 Zoltán and his 5 year-old brother were placed in a Montessori boarding school in Vienna. This environment emphasized individualized learning and autonomy. It was here that Zoltán encountered Montessori educational tools that later influenced his own educational theories.

Raymond Duncan's Commune in Nice and Paris (France): Zoltán's family (except for the father) joined Raymond Duncan's commune in France. This commune was characterized by a blend of artistic, philosophical, and alternative beliefs, with a focus on self-expression and a rejection of traditional societal norms. Zoltán experienced communal living during this period, which had a lasting impact on his attitude to material goods.

Public school in Pápa (Hungary): At this school, Dienes encountered difficulties in adjusting to the Hungarian language and culture, as well as the traditional Hungarian educational system, which also included religious education. Additionally, the sudden shift in his mother Valéria's religious beliefs and practices, influenced by her Catholic conversion, added a layer of complexity to his upbringing. His experiences were marked by a sense of anxiety and recurring nightmares. Nonetheless, this one-year period introduced him to the intricacies of different faiths, eventually shaping his unique worldview.

Új Iskola (New School) in Budapest (Hungary): from the age of 7 to 10 Zoltán attended this progressive school in Budapest, which emphasized creative thinking, individual interests, and experiential education. Here, he further developed his intellectual capabilities and creativity.

Piarist Secondary School in Budapest (Hungary): Zoltán enrolled in this Catholic school, which provided a traditional yet high-quality education. It marked a shift towards a more structured and disciplined learning environment, contrasting with his earlier experiences.

Trans-European Peregrination: During vacations, Zoltán and his brother travelled extensively with their father, exposing them to diverse European cultures and languages. These summer holiday trips enabled the father to foster a vibrant connection with his sons, fostering their intellectual curiosity and offering them a positive male role model.

Dartington Hall School, Devon (United Kingdom): Zoltán's experience at this progressive English school solidified his preference for child-centered, joyful learning. The school encouraged individuality, creativity, and independence, aligning with his prior educational experiences.

In addition to outlining the various cultural contexts, it is essential to shed light on Zoltán's familial social connections. Despite his parents' divorce when Zoltán was just 3 years old, his relationship with both of them remained close and intimate throughout his childhood. Both parents worked hard to ensure he received the best education, learned languages, explored neighbouring countries and their cultures, and gained insight into both progressive thinking and traditional religious values. Not only were his parents emotionally supportive, but they also served as intellectual role models, embodying the ideal of highly educated scholars. Throughout his childhood, Zoltán maintained a strong and deep connection with his brother Gedeon, with whom he mutually supported and engaged in all activities, thus never experiencing a sense of abandonment. Additionally, both maternal and paternal relatives were consistently available for support when needed. This rich and nurturing social network transcended the financial and existential challenges that were ever-present during Zoltán's upbringing.

In my prior research, I examined how the above distinct cultural contexts influenced the development of Dienes's character. I concluded that as an adult,

Dienes displayed exceptional tolerance for diverse worldviews, cultures, and lifestyles. He consistently embraced the chance to learn from differences, fostering an environment of love wherever he resided and taught. In this study, I chart the course of Dienes's upbringing again, emphasising the cognitive tools he acquired from various cultures that played a pivotal role in shaping his intellectual development. My research centres on childhood because the Vygotskian theory, which I've selected as the explanatory framework, is particularly concerned with child cognitive development.

# The Dienes method of teaching mathematical concepts

The Dienes method breaks away from traditional efforts to improve numeracy skills and instead aims to develop mathematical thinking by constructing mathematical structures into the minds of students. It is based on either play activities that embed abstract mathematical concepts or playful manipulation of specially designed mathematical tools that embody abstract mathematical structures. In the development of mathematical concepts, the Dienes method initially encourages implicit learning through play. The representation of an abstract concept provided by the teacher can take various forms, such as a series of movements or dance choreography, being embedded in a melody or story, or encoded in play sets such as the Dienes blocks or the Logic blocks. By being exposed to multiple embodiments of the same abstract structure, the learner eventually discovers the nature of the hidden concept in a moment of insight. Insight is a crucial turning point in the learning process, where the mathematical structure that was previously outside the learner's mind is brought inside as its own mental representation. The mental image thus formed must then be transformed into explicit knowledge, which is also achieved through some further playful explicit learning. Upon completing the learning process, the once hidden abstract mathematical structure embedded within the diverse games not only emerges but also attains a high level of elaboration in the learner's mind (Dienes, 1973; 1989). Throughout the learning process, the Dienes method expertly transitions between domains, transforming the abstract into the concrete and the implicit into the explicit, and vice versa. It embodies a constructivist approach to learning and implies the later formulated ideas of distributed and embodied cognition. (Bálint, 2015; Benedek, 2018b; Benedek & Tuska, 2020; Dienes, 2007).

#### Aims of the study

The way of unfolding and the development of intellectual skills is a central issue when one intends to understand the cognitive performance of an extraordinary individual, who was a recognised scholar of his time and eventually developed a new method of teaching mathematics. Although widely recognised for groundbreaking discoveries in mathematical learning theory, less examined are the formative early experiences that equipped Zoltan Dienes with the cognitive tools and conceptual orientations underlying his method. Tracing Dienes's cognitive and conceptual development from a young age contextualises his later theorising in a more grounded manner.

Originally, psychobiography heavily depended on psychoanalysis (Elms, 1994) and centred on probing intrapsychic events when portraying the subject's life journey. However, psychobiographical research has progressed, embracing a broader theoretical foundation (Ponterotto, 2014; Mayer et al. 2021) and methodological (Mayer & Kovary, 2019) approach. Psychobiographical research additionally expanded its scope to non-WEIRD, transcultural and transdisciplinary perspectives (Mayer et al. 2023) and tends to include theories and methods outside psychology to broaden its interpretative set (Mayer, 2023). The recognition of the individual's integration into their historical context has been reinforced, and the investigation of their social relationships system presents exciting prospects for psychobiographical research, employing the methodologies of network science (Rab, 2019; 2021a; 2021b; 2023). Despite these advancements, the investigation of intellectual development has received limited attention. In my current study, I aim to incorporate the cognitive developmental aspect into psychobiographical research.

My aims are dual: (1) to enrich the methodological set of psychobiography by incorporating the Vygotskian sociocultural theory, which provides a deeper understanding of how culture impacts the intellectual growth of knowledgeable individuals, and (2) to contribute to a better comprehension of the interconnection between the life and thinking of Zoltán Paul Dienes.

#### **Methods and sources**

The range of cognitive tools offered by each culture is limitless and attempting to explore them exhaustively would be a hopeless endeavour. As such, my aim is not to provide a comprehensive inventory but rather a rough overview of the growing cognitive toolbox of Dienes during his childhood.

To achieve this goal, I developed a simple taxonomy that categorises the cognitive tools present in each culture, taking into account the cultural and technological development of the early 20th century. The taxonomy comprises four main categories: Cultural, Communication, Educational. Information & knowledge tools. I then arranged the cultural sites young Dienes encountered in chronological order. Subsequently, I created four tables, one for each main cognitive tool category, listing the subcategories in the first column and the cultural sites in chronological order at the head row. The presence of each tool was indicated with a "+" mark, with multiple "+" denoting frequent exposure. This systematic approach allowed me to track the development and extension of Dienes's cognitive toolset over time, providing insights into his cognitive growth and shedding light on the origins of the principles his later theory of mathematical education.

My primary sources consist of Dienes's comprehensive autobiography (Dienes, 1999) written at the age of 82, and two interviews with his mother, Valéria Dienes (Dienes V., 1983; Szabó, 2001) conducted in her later years. These texts offer valuable insights into Dienes's formative years and help identify various cognitive tools that contributed to his intellectual development. Both Zoltán and Valéria Dienes, in their elderly wisdom, provide reflections on the significance and origins of ideas, life events, and experiences which considerably helped my interpretative Furthermore, I rely on two of Dienes's books (Dienes, 1973; 1989) in which he discusses his theory. Alongside these primary sources, my secondary sources encompass studies on Dienes's work and theory, as well as research on other prominent family members.

#### **Short family history**

Zoltán Paul Dienes is a member of the esteemed Hungarian Dienes family, renowned for producing a lineage of accomplished individuals, including clergymen, solicitors, doctors, scientists, and scholars across multiple generations. His parents were highly educated and multilingual people, pioneers of progressive movements in early 20th-century Budapest, advocating for reform pedagogy, feminism, leftist ideas, and even Communism. Both earned their doctorate degrees in mathematics from the University of Budapest in 1905 and pursued further studies at the Sorbonne in Paris (Dienes V., 1983).

Paul Dienes (1882-1952), the father, was a dedicated high school teacher specialising in mathematics and physics. With an additional doctorate degree from the Sorbonne, he expanded his career as a university professor and mathematician. Despite participating in the Hungarian Communist Revolution of 1919, he later distanced himself from left-wing movements and embraced freethinking liberalism. Forced to leave Hungary after the revolution's fall, he found his new home in the United Kingdom, where he pursued a successful career as a university professor. In his mathematical pursuits, he focused primarily on the study of Taylor series (Benedek, 2018b; Dienes V., 1983).

Valéria Dienes (née Valéria Geiger), the mother (1879-1978) of Zoltán Paul Dienes, was a remarkable philosopher, psychologist, and dance theorist. During her studies in Paris (1906-1909), she became an enthusiastic follower of Henri Bergson, whose ideas greatly influenced her theory of movement art, which developed between 1912-19. Another significant influencer of her theory was Raymond Duncan, known for his interpretations of Greek dances from skeletal paintings (Benedek, 2018b; Boreczky & Fenyves, 2017; Dienes, 1983; Szabó, 2001). Valéria attended his Greek gymnastics classes and felt compelled to "systematize these ideas" (Dienes V. 1983), 57). After leaving Hungary in 1919, the divorced Valéria spent a whole year with Duncan at his commune in Nice, where she felt disillusioned by his reluctance to develop his movement system further. "Raymond did not even think of further developing the movement - he said that what he taught in movement could not be added to" (Dienes V. 1983, 59).

Returning to Hungary in 1923, Valéria continued her philosophical work while supporting her children through teaching dance. She further refined Orchestics, a theory of the "speech of the body" that included four interdependent levels: plasticity, rhythm, dynamics, and symbolism (Dienes V. 1983; Boreczky, 2021). These levels exemplified the implicit process of conceptualisation based on movement, reflecting Valéria Dienes's intellectual philosophy, which integrated Bergsonian thought along with other ahead-of-its-time ideas. This philosophy incorporates intuition, implicit learning, body-awareness, a theory of consciousness, the power of imagination, and even the ideas of distributed and embodied cognition. Valéria was convinced of the healing power of movement for both body and soul, and even speculated about movement-induced brain plasticity (Dienes V., 1983). Upon her return to Hungary, she converted to Catholicism and became an influential Catholic thinker.

Gedeon Dienes (1914-2005), the elder brother of Zoltán Paul Dienes, was a multifaceted individual with talents in dance, choreography, dance history, criticism, and linguistics. Their closely intertwined childhoods and upbringing in the same cultural contexts imply that much of what applies to Zoltán is also relevant to Gedeon. However, their paths diverged in the 1930s. While Zoltán pursued an international career in mathematics, moving to the UK for further studies, teaching at various universities worldwide, and freely touring around the globe, his equally gifted older brother remained in Hungary, where he was unable to fully realise his diverse talents due to the socialist turn of the history of the country. Gedeon Dienes, in addition to his varied talents, possessed an extraordinary linguistic aptitude, speaking fluently in over 20 languages during his adulthood. He also took on the vital role of preserving and passing on Valéria Dienes's Orchestical heritage. Despite their distinct life trajectories, the bond between the brothers was preserved until Gedeon's death in 2005 (Dienes, 1999).

The family's life underwent a significant transformation following the fall of the Communist Revolution in 1919. Seeking refuge, they relocated to Austria, where the two boys were admitted to Montessori Kinderheim, a nursery school in Vienna. Subsequently, the parents divorced, leading to divergent paths. The mother and the boys ventured to France, residing in Raymond Duncan's commune.

Life in Raymond Duncan's commune was characterised by a strong emphasis on simplicity, communal living, and a dedication to traditional Greek culture. The community sought to live harmoniously, fostering creativity through various artistic endeavours, craftsmanship. Life in the commune was marked by a delicate balance between his inspiring leadership and the challenges posed by his dictatorial tendencies (Bálint, 2021; Dienes, 1999; Szabó, 2001).

After approximately two years abroad, Valéria and the two boys returned to Hungary. Meanwhile, the father remarried and settled in the UK to pursue his career. Despite the separation, both parents actively participated in their children's upbringing: the mother cared for them during the school year, while the father looked after them during summer holidays, ensuring their intellectual development was profoundly influenced by both (Dienes V. 1983; Dienes, 1999).

#### Cognitive tools provided by diverse cultures

In what follows, I will evaluate the cognitive tools that different cultures have offered the child Dienes. This categorisation of cognitive tools which I made

up myself, is not exhaustive, since the classification can vary depending on the context and perspective. Cognitive tools are dynamic and evolve with changes in technology, culture, and societal needs. The following categorisation aims to showcase the diverse range of cognitive tools present in various cultural settings during the 1920s and 30s.

Communication tools facilitate interaction, expression, and cultural transmission. Cultural tools help preserve and transmit cultural knowledge and facilitate social interactions. Educational tools support learning and intellectual development. Information and knowledge tools serve as external aids that extend human cognitive capacities, fostering improved learning and problem-solving outcomes. In the following discussion, I will begin by exploring Communication Tools, followed by Cultural and Educational Tools, and finally, I will cover Information and Knowledge Tools.

#### **Communication tools**

#### Language

Language is one of the most fundamental cognitive tools, "the tool of tools" (Vygotsky, 1962). I have marked each language Dienes acquired and those taught at school with a "+" sign and additionally, I named each to enhance clarity. For Summer Holidays, I have indicated the languages Dienes learned on the spot or reported that he was actively learning with his father's encouragement.

Dienes's cognitive development was deeply influenced by his exposure to multiple languages throughout his upbringing. His first language, Hungarian, laid the foundation for his early cognitive style, shaping how he perceived spatial relations and causality. At the Montessori Kinderheim in Vienna, he was immersed in German, which expanded his conceptual repertoire and cognitive flexibility.

Moreover, the acquisition of French at Duncan's commune further expanded his linguistic repertoire and exposed him to different ways of categorising experience, influencing his tendency towards analogical and metaphorical thought. It is worth noting that both ancient and modern Greek were also present in the commune: the children's daily routine included singing Greek songs. "These were, I believe in modern Greek, and we all learned to sing the catchy tunes, vocalising the words as best we could, but Duncan forgot to tell us what the words meant and to this day, I remember some of the words but have absolutely no idea what they mean!" (Dienes, 1999,

26) Being exposed to Greek songs at a young age, even without understanding the meaning could further enhance his multilingual flexibility as well as facilitate code-switching in the future.

Returning to Hungary at age six was a pivotal phase in Dienes's cognitive development. Over the next eight years, attending various schools moulded his linguistic repertoire and conceptual formation. French was spoken predominantly at home, while Hungarian served as the primary language at school. Additionally, German and Latin were introduced as foreign languages. Moreover, the parent's influence further encouraged his linguistic curiosity. Dienes vividly recalls receiving multiple grammar books on his birthday, including Romanian and Slovak ones, fuelling his eagerness to explore and understand foreign languages (Dienes, 1999).

As if his multilingual journey wasn't remarkable enough, Dienes and his brother embarked on the creation of their own artificial language, the Sazla language, at the age of eight (Dienes, 1999). This endeavour showcased Dienes's proclivity for logical and creative thought, even at such a young age, foreshadowing his future work on mathematical pedagogy. Notably, the Dienes family's linguistic inclination extended beyond the two boys. Their father, Paul Dienes, shared their fascination with artificial languages. He had extensively researched and analysed various artificial languages in his 1914 paper, "The Problem of Auxiliary International Language," expressing support for Ido over Esperanto (Dienes P., 1914). Paul Dienes's involvement in the development of Ido, as part of a project led by the French mathematician and linguist Louis Couturat, highlights the family's deep interest in linguistic exploration (Benedek, 2018a).

During the summer holidays, the Dienes brothers travelled around Europe with their father from ages seven to fourteen which provided ideal context for picking up new languages. They easily acquired Italian and learned some other local languages. Learning English upon relocating to the UK at age 15 was challenging yet fruitful for Dienes. The need for independent functioning acted as a catalyst for learning. The process took about a year, with support from friends and the authentic linguistic environment, making English his first language and providing access to a new cultural toolkit (Dienes, 1999).

This exposure to multiple languages not only expanded his conceptual horizons but also paved the way for his future work in mathematical pedagogy, showcasing his natural aptitude for logical and mathematical thinking.

#### Writing systems

Writing systems serve as cognitive tools that enhance human memory, communication, and knowledge transfer by externalising information, enabling abstract thinking, and facilitating complex learning processes. Writing systems enable individuals to represent spoken language in a symbolic form. Newly learned writing systems were indicated in the table with a "+" sign and additionally named to enhance clarity. According to Table 1, Dienes was exposed to two different writing systems (Latin and Gothic), as well as several different alphabets, including Hungarian, German, Latin, Slovakian, Romanian, Italian, and English. Additionally, at the Piarist Grammar School, Ancient Greek was used in mathematics and physics to signify constants or measures. Exposure to multiple alphabets must have enriched Dienes's cognitive, linguistic, and cultural development. It broadened his horizons, promoted cognitive flexibility, and nurtured an appreciation for global linguistic diversity, ultimately contributing to his overall growth and understanding of the world.

#### Readina

Reading is the cognitive tool through which we extract information from written texts. Different languages favour different reading modes and strategies, so I have indicated the acquisition of this tool with a "+" sign for each newly acquired/learned language to read on, additionally naming these languages as well in the corresponding cells.

Learning to read and write in French was Dienes's initiation into decoding texts. This experience fostered symbolic thinking and the ability to manipulate and interpret abstract symbols. Dienes's reading ability allowed him to explore stories, and he became a passionate reader with an almost insatiable thirst for tales (Dienes, 1999, 25-26). As Dienes learned to read and write in other different languages, including Gothic German and Ancient Greek, he delved into the world of symbolic representation.

#### Writing and creating texts

Writing and creating texts are cognitive tools that facilitate the externalisation of thoughts and ideas, allowing for proper structuring of a text. Schools in the early 20th century placed great emphasis on the development of these skills, so I have marked all

Table 1. Communication tools

Communication tools									
Macro level culture	Hungary	Austria	France	Hungary	Hungary	Hungary	Europe	UK	
Micro level culture	Budapest family	Vienna Montessori Kinderheim	Nice, Paris Duncan's commune	Pápa public school	Budapest New School	Budapest Piarists' Grammar School	summer holidays	Devon Dartington Hall School	
Language	+ Hungarian	+ German (Austrian)	+++ French Greek (anc. and modern)	+++ Hungarian German (stand.) Latin (anc.)	++ German (stand.) Latin (ancient)	++ German (stand.) Latin (anc.)	++++ German (stand) Italian French English	+++ English (stand.) English (med.) Latin (med.)	
Writing systems			+ French	+ Hungarian Latin	+ Gothic	+ Greek	+ Italian	+ English	
Reading			+ French	+++ Hungarian Latin German	++++ Gothic Slovakian Romanian Sazla		+ Italian	+ English	
Writing and creating texts			+	+	+	+	+	+	
Oral traditions and story-telling	+	+	++	+	+	+	++	+	

schools with a "+" sign where these tools were certainly actively used and developed.

As Dienes learned to write and create texts, he engaged in the process of expressing his thoughts and ideas through language as well as structuring stories according to narrative rules. Writing allows to express creativity and imagination. Fictional stories, essays, and poems provide an outlet for self-expression and encourage innovative thinking. Throughout his life, Dienes produced numerous scientific articles and books, as well as a book of poems. In addition, he created stories for mathematical teaching purposes.

#### Oral traditions and storytelling

Oral traditions and storytelling serve as cognitive tools that enrich language development, memory, imagination, cultural understanding, and social skills. They include everything from nursery rhymes to oral history accounts. Storytelling in particular benefits us by engaging emotions, enhancing memory retention, and facilitating meaningful comprehension of information through narrative structures.

In all cultures, members are provided with an abundance of oral traditions, which I have indicated with a "+" in the corresponding cell of Table 1. Double "+" indicates exposure to more than one oral tradition at a time or the abundance of stories he was exposed to.

In his early years, Dienes encountered Hungarian folktales, songs, and customs. At the Montessori school, he learned nursery rhymes, including little stories. At the Duncan commune, he was immersed in both French and Ancient Greek stories. This exposure to oral traditions and storytelling structured how he imagined causality, agency, and possibility.

In summary, Dienes's cognitive development was significantly influenced by the diverse communication tools that he encountered throughout his special cultural experiences. Language provided the foundation for his thought patterns, while writing systems and reading enhanced his symbolic thinking and engagement with literature. Creating texts and engaging with storytelling facilitated expression and understanding of concepts. The immersion in various oral traditions enriched his understanding of cultural beliefs and values. The combination of these cognitive tools formed the basis of Dienes's intellectual journey and his contributions to the fields of mathematics and pedagogy.

#### **Cultural tools**

#### **Community practices**

Community practices and traditions, as cognitive tools, encompass shared customs and social behaviours that enhance collective learning, memory retention, and cultural identity, fostering a sense of belonging and facilitating knowledge transfer across generations.

Dienes's educational journey exposed him to diverse community practices in each cultural context (see Table 2.). In comparing these diverse cultural sites and

Table 2. Cultural tools

			(	Cultural tools				
Macro level culture	Hungary	Austria	France	Hungary	Hungary	Hungary	Europe	UK
Micro level culture	Budapest family	ViennaMonte-ssori Kinder-heim	Nice, Paris Duncan's commune	Pápa public school	Budapest New School	Budapest Piarists' Grammar School	summer holidays	Devon Darting-ton Hall School
Community practices and traditions	+	+	+	+	+	+	+	+
Rituals		+	++	+	+	++		
Musical instruments	++	+	+		++			++
Visual and fine arts tools	++	+	++		++	+	+	++
Cultural artefacts and artworks	++	++	++	++	++	++	++	++

<sup>+:</sup> present; ++: abundant.

their respective community practices and traditions, it's important to note that each environment has unique strengths and focuses on different aspects of cognitive development. The Dienes parents were highly educated, and their lives were interspersed with philosophy, science, and the arts. They highly valued books and had built up a unique library at home. Their daily practices could serve as cognitive tools for their children for learning and understanding the world.

In the Montessori schools, Dienes experienced self-directed learning, collaboration, and the development of social skills, nurturing his curiosity and passion for exploration. Duncan's commune prioritised communal living, shared responsibilities, and group discussions, fostering cooperation and a sense of collective identity.

Upon returning to Hungary, attending the public school and Piarists' Grammar School exposed him to community practices centred around regular religious activities, strict rules, and intellectual pursuits, shaping his critical thinking and character development. In contrast, the progressive New School, akin to Montessori principles, and the subsequent school in the UK, Dartington Hall, emphasised experiential learning, individuality, outdoor activities, and emotional intelligence, providing cognitive tools for well-rounded development.

Through these diverse cultural sites, Dienes gained unique cognitive tools, each contributing to his growth and understanding of the world around him.

#### Rituals

Rituals and ceremonies are cognitive tools that contribute to the formation of cultural identities and

the transmission of cultural values and beliefs. The sites that were most likely to be rich in rituals were probably the Duncan Commune and the Piarist Grammar School (see Table 2.). Duncan believed in the spiritual and moral benefits of Greek art and culture. He sought to revive ancient Greek ideals of balance and harmony through his art, stressing the ceremonial and ritual aspects of dance (Dienes V., 1983).

The Catholic school stands out from the rest in terms of the frequency of its religious ritual practices, such as daily Mass, confession, and ceremonies (Dienes, 1999). The exposure to rich rituals in Duncan Commune and Piarist Grammar School may have provided young Dines with a sense of cultural identity, aesthetic appreciation, and an understanding of the importance of ceremony and tradition

#### **Musical instruments**

Musical instruments serve as cognitive tools for composing and performing music, promoting creativity and expression in the arts. The landscape concerning access to musical instruments exhibits considerable diversity. Valéria Dienes was an educated and passionate pianist (Dienes V. 1983). Additionally, the Montessori school and commune environments also provided a stimulating influence on the cultivation of musical abilities. It is probable that further exposure to music and instruments continued to manifest at the New School in Budapest and Dartington Hall school. The access to musical instruments could foster musical skills, creativity, and aesthetic appreciation,

while also contributing to overall cognitive development and emotional expression.

#### Visual and fine art tools

Visual and Fine Arts Tools: Tools used in visual arts, such as paintbrushes, palettes, and sculpting tools, were cognitive instruments for artistic expression and representation. Throughout the various contexts of Dienes's formative years, visual and fine art tools held a pervasive presence, particularly evident in the art-loving ambiance of his home in Budapest and the alternative educational institutions that placed a strong emphasis on art education. Notably, the Duncan commune provided Dienes with a unique opportunity to observe and engage with the impressive artistic creativity of its members, allowing him to actively explore various artistic tools and expressions (Dienes, 1999). The exposure to art across these diverse settings likely played a crucial role in fostering Dienes's cognitive and emotional development, promoting creativity, aesthetic appreciation, and a deeper understanding of self-expression.

#### **Cultural artifacts and artworks**

Cultural Artefacts and Artworks: Cultural artefacts, historical objects, and artworks are cognitive tools that convey historical narratives, cultural heritage, and artistic expressions. The exposure to cultural artefacts and artworks during Dienes's upbringing was exceptional, stemming from the diverse cultural environments in which he was raised and his family's elevated appreciation for aesthetics. This exceptional access provided him with a wide array of stories, pictures, sculptures, musical pieces, and other artworks. The abundant exposure to diverse Cultural Artefacts and Artworks throughout his formative years contributed significantly to Dienes's cognitive and emotional development, fostering an enriched imagination, cultural awareness, which could have had a profound impact on his intellectual journey.

#### **Educational tools**

#### **Educational materials**

Educational materials such as textbooks, chalkboards, and educational toys, as well as graphs, tables, and timelines, serve as cognitive tools and are commonly used in classrooms to facilitate learning and understanding of various subjects. Throughout his educational journey, Dienes was consistently exposed to diverse educational tools and materials that played a pivotal role in shaping his cognitive and intellectual development, as well as nurturing his passion for learning and exploration. At the Montessori school as well as the progressive New School Dienes had access to diverse educational tools designed to enhance sensory abilities and foster self-directed learning, which likely contributed to the development of his innate curiosity and love for exploration. The well-equipped Hungarian school provided Dienes with educational materials such as globe models, maps, and telescopes, enriching his understanding of the world and promoting spatial and scientific thinking. Dartington Hall provided an enriching environment with arts, crafts, and outdoor activities (Dienes, 1999), fostering emotional intelligence, social skills, and an appreciation for experiential learning in Dienes.

#### Games and puzzles

Games and puzzles, including board games, puzzles, and intellectual challenges, are cognitive tools that foster problem-solving skills, critical thinking, and strategic planning. All cultures are exceptionally rich in these. I have marked with double "+" the cultural sites that were most likely to offer Dienes more of these cognitive tools than the average. As shown in Table 3, these cultural sites are the family home, alternative schools, and European routes. Notably, during these travels, his father introduced him to mathematical equations in the form of intriguing riddles. (Dienes, 1999) Exposure to games and puzzles offers a wide range of cognitive benefits, including improved

Table 3. Educational tools

Educational tools									
Macro level culture	Hungary	Austria	France	Hungary	Hungary	Hungary	Europe	UK	
Micro level culture	Budapest family	Vienna Monte-ssori Kinder-heim	Nice, Paris Duncan's commune	Pápa public school	Budapest New School	Budapest Piarists' Grammar School	summer holidays	Devon Darting-ton Hall School	
Educational materials		++	+	+	++	++	+	++	
Games and puzzles Measurement and calibration tools	++	++ +	++	++	++ +	++	++	++	

<sup>+:</sup> present; ++: abundant.

problem-solving skills, cognitive flexibility, memory, spatial reasoning, pattern recognition, creativity, and social skills.

#### Measurement and calibration tools

Tools used for measurement, such as rulers, clocks, thermometers, and scales, serve as cognitive tools for quantifying and understanding the physical world. These educational tools were all found in the various well-equipped educational institutions that Dienes attended as indicated in Table 3. Being exposed to measurement and calibration tools during formative years provides numerous cognitive benefits, including improved spatial reasoning, mathematical understanding, problem-solving skills, logical thinking, and data analysis abilities. Additionally, it enhances fine motor skills, encourages scientific inquiry, and fosters cross-disciplinary applications while equipping individuals with valuable practical life skills.

#### Information and knowledge tools

#### Scientific instruments

Scientific instruments, such as telescopes, microscopes, and laboratory equipment, are cognitive tools for conducting experiments and advancing scientific knowledge. In the 1920s, Hungarian schools were well-equipped with these tools. The family, whose life was intertwined with science, also had access to them. Table 4 displays the presence of scientific instruments without specifying any cultural sites.

In his autobiography, Dienes explicitly recalls how he and his brother naturally utilised tools like these in conjunction with their mathematical expertise. Upon entering school in Hungary, Dienes was first introduced to the world of scientific instruments. After relocating to Budapest, their first home was a small house nestled in the midst of a spacious garden. It served as the inspiration for the Dienes brothers to systematically map it out. "We measured all the paths, the distances between the copes and the length and the width of the entire garden by pacing out the distances. It took us several weeks of work but eventually, we came up with a fairly accurate map, using a definite scale, whose ratio I cannot remember now. I suppose this might have been our first introduction to geometry" (Dienes, 1999, 36-37).

Exposure to scientific instruments in formative years provides valuable cognitive benefits, including hands-on learning, observational skills, cross-disciplinary understanding, and appreciation for scientific methodology, and nurtures creativity and innovation.

#### Memory techniques

Mnemonics and memory techniques are cognitive tools used for enhancing memory and recall of information. In the early twentieth century, the educational practice of providing children with ample material to memorise for enhancing their memory skills remained prevalent. Traditional schools, like the public school in Pápa and the Piarist Grammar School, presumably heavily relied on this practice, whereas progressive schools likely employed different approaches. This difference is indicated in Table 4 where the number of " +" marks reflects this contrast.

Dienes's earliest memory of having something to remember comes from the time when the Duncan commune moved from Nice to Paris. The children had to learn the new address in case they got lost. "I still remember the address clearly; it was rue de Colisée trente-quatre." (Dienes, 1999, 24) Upon returning to Hungary Valéria hired a private Latin teacher who taught the two boys Latin grammar in rhymes. "Gedeon and I would recite these verses as we walked along the streets of Budapest, to the

Table 4. Information and knowledge tools

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Information and knowledge tools									
Macro level culture	Hungary	Austria	France	Hungary	Hungary	Hungary	Europe	UK	
Micro level culture	Budapest family	Vienna Monte-ssori Kinder-heim	Nice, Paris Duncan's commune	Pápa public school	Budapest New School	Budapest Piarists' Grammar School	summer holidays	Devon Darting-ton Hall School	
Scientific instruments				+	+	+	+	+	
Memory techniques		+	++	++	+	++	+	+	
Cognitive strategies and heuristics	++	++	++	++	++	++	++	++	

<sup>+:</sup> present; ++: abundant.

astonishment of the passers-by who naturally could not make out what these crazy boys were saying!" (Dienes, 1999, 39) Apart from these, numerous other memory techniques might have contributed to Dienes's verbal recall, which he did not detail in his autobiography.

Memory techniques, as cognitive tools, are beneficial for enhancing memory retention, recall, and organisation of information. They possibly facilitated Dienes's later mathematical thinking through better retention, recall, and application of mathematical concepts, leading to a deeper understanding and appreciation for the subject.

#### Cognitive strategies and heuristics

Problem-solving strategies, algorithms, and heuristics are cognitive tools that enhance memory and facilitate decision-making. The Dienes parents consistently fostered an inspiring learning environment, motivating their children through their own example and providing enriching materials to nurture intellectual growth. Additionally, the educational institutions Dienes attended upheld the highest academic standards and offered valuable cognitive strategies and heuristics to enhance learning. The corresponding cells of Table 4 indicate the abundance of these cognitive tools throughout the cultural contexts.

Dienes's autobiography brims with enthusiasm for learning, particularly for mathematics. His knowledge quickly surpassed school requirements, and he was fortunate to have mathematician parents and access to books written by mathematicians. In his own words, Dienes candidly expressed, "The curriculum we were doing at school I found trivial, for everything seemed rather obvious and easy, not much of a challenge, so during the mathematics lessons I often did not listen to what was going on but tried to amuse myself by solving various differential equations." (Dienes, 1999, 62) Despite feeling disengaged in maths classes, Dienes's passion and talent did not go unnoticed. His mathematics teacher at the Piarist grammar school, Zoltán Ferenczi, himself a mathematician, recognised Dienes's exceptional abilities and took the initiative to offer extra teachings and engage him in discussions about mathematical problems. Zoltán Ferenczi's instrumental role in nurturing and cultivating Dienes's talents played a crucial part in shaping his remarkable mathematical journey.

The constant exposure to supportive learning environments and effective cognitive strategies played a significant role in shaping Dienes's

development, paving the way for his successful career as a mathematician, where he utilised these foundations to contribute significantly to the field of mathematics education.

#### Limitations and future research

This study explores the cognitive tools that influenced the cognitive development of Zoltán Paul Dienes, focusing solely on social and individual learning and ignoring the role of genetic heritage. Furthermore, it remains unclear how much Dienes received his theoretical ideas from his highly educated parents in ready form versus how much he developed them through the aid of acquired cognitive tools. This distinction is challenging to discern, making it a potential limitation of the study.

Another limitation stems from the impossibility of identifying and considering the entirety of existing cognitive tools. Therefore, this study employs a more or less arbitrary selection of cognitive tools, which may not encompass the full spectrum of tools that could have influenced the cognitive development of the individual.

Future research is essential to delve deeper into the complexities of cognitive development, including a broader range of cognitive tools and considering both genetic influences, and the rate of the incorporated ideas to provide a more comprehensive understanding of Dienes's cognitive journey and the origin of his theoretical ideas.

#### **Discussion**

Utilising Vygotskian theory on cognitive tools in psychobiographical research enhances our comprehension of intellectual growth, as it grounds it in the concrete details of an individual's life. Dienes's experiences serve as a prism, revealing how diverse cognitive tools from various cultural contexts shape an individual's cognitive development.

In the specific case of Dienes, these multicultural experiences nurtured his later inclination to view knowledge as personally constructed rather than passively received. This perspective was key to his theory of self-generated mathematical insight. Immersion in multiple languages furnished Dienes with linguistic cognitive tools like flexible thinking patterns, comfort with abstraction and precision in reasoning - all abilities integral to advanced mathematics. His multilingualism enriched his conceptual power. Exposure to progressive and experimental education from an early age trained Dienes's mind to be curious, creative, self-motivated and comfortable with unpredictable experiences. These skills served him well as an innovator in mathematics education. The combination of progressive and more traditional schooling balanced intuitive and analytical approaches to knowledge, training Dienes to value both experimentation and rigour equally. This ecumenical disposition served him well as an interdisciplinary thinker.

Dienes's innovative mathematics teaching method was significantly shaped by the cognitive tools provided through his formative early experiences. By focusing on playful and embodied experiences, the method made mathematical concepts more tangible and accessible to learners. The emphasis on insight and transformation from implicit to explicit knowledge allowed for a deep understanding and internalisation of mathematical structures. The constructivist approach ensured that learners actively engaged in the learning process, making it more meaningful and personally relevant. Altogether, these elements contributed to the success and effectiveness of the Dienes method in developing mathematical thinking in students.

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#### **Ethical approval**

Ethical considerations have been applied in this research.

#### **Disclosrue statement**

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