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Laboratory Investigation

Mathematical Determination of the Tibial Insertion of the Patellar Tendon Using Computed Tomography Images

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ABSTRACT Misalignment of the extensor apparatus is an essential factor in impairment of the patellar-femoral joint. This may be partly or entirely responsible for patellar dislocation or lateral patellar-femoral arthrosis. One surgical method to correct the pathology is medial transposition of the patellar ligament on the tibial tuberosity (anteriorly or posteriorly, distally or ventrally). These interventions correct misalignment of the extensor apparatus relative to statistical norms. We propose a mathematical method based on the use of computed tomography (CT) images to determine the ideal tibial insertion for the patellar tendon. This method is based on biomechanical modeling and the use of equipressure criteria. It is the first step in allowing the use of mathematics to model correctly tibial insertion of the patellar ligament, an entirely new development. This is important because it will allow surgeons greater accuracy in distal correction of extensor apparatus misalignments.

INTRODUCTION

Patellar instability and distal femoropatellar arthrosis have several etiologies. One of them is extensor apparatus misalignment. Let us consider that the vector representing the quadriceps and the pulley made by the femoral trochlea are both fixed. The misalignment of the extensor apparatus presents a position defect on the anterior tibial tuberosity (ATT) at the tibial insertion of the patellar tendon. Notably, for a given patellar tendon, the tibial insertion site can either be too external or too proximal. It is easy to use the patellar index of Caton and Deschamps to normalize the position of the ATT along the proximal/distal direction. It is more often debated and more difficult to determine the length of the inner transposition of an obviously excessively external ATT.

Currently, two methods exist to measure misalignment in the coronal plane: Anglo-American surgeons use the quadriceps angle (i.e., the angle made by the quadriceps tendon with the patellar tendon), whereas French physicians use the TAGT measure [tuberosité antérieure-gorge de la trochée (ATTG), which in English stands for anterior tuberosity-trochlear groove]. This measure is the distance in millimeters between two parallel vertical lines projected on the coronal plane of the ATT and perpendicular to the tangent joining both posterior femoral condyles. One vertical line intersects the top of the tibial tuberosity, and the other intersects the proximal part of the trochlear groove (Figs. 1, 2). It is still debated whether the ATTG must be measured with the knee in full ex-
tension or at a 15° angle of flexion. We measure the ATTG with the knee in full extension to enhance reproducibility. It is assumed that an inner transposition of an ATT that is too external tends to "normalize" the ATTG to a value between 10 to 15 mm (in full extension). In this paper, a different mathematical approach to this problem is offered, based on biomechanical hypotheses described in the next section.

In this model, practical considerations and simplicity dictate that 1) we consider geometric features with the knee in full extension and 2) we deal only with mathematical determination of the inside simple anterior tibial tuberosity transposition (ATIT), without Maquet or anti-Maquet effects and without proximal or distal displacement (associated parasites). In later sections, we describe mathematical modeling of the hypotheses, give the data obtained using computed tomography (CT) images for this model, give the mathematical calculus of the transposition, and discuss our results.

BIOMECHANICAL STUDY

Notation

In the general case, let the force created by the quadriceps be \( F_q \), let the force created by the patellar tendon be \( F_p \), and let the knee flexion angle be at a defined position \( \theta \). \( R \) is the sum of those two forces (see Fig. 2):

\[
R = F_p + F_q
\]

The study of the spatial direction and position of \( R \) in a known reference coordinate system will give a numerical indication for the misalignment and helps to quantify the surgical maneuver in function of the effect desired. The resultant \( R \) exerts itself on the contact sites between the two trochlear cheeks and the patella. This area is divided into \( S_i \) (the contact area on the internal trochlear cheek) and \( S_e \) (the contact area on the external trochlear cheek). Let \( P \) be the plane perpendicular to the proximal part of both \( S_i \) and \( S_e \). When the knee is in full extension, \( P \) is the horizontal plane (see Fig. 3). \( R_p \) is the projection of \( R \) on the plane \( P \), and \( F_i \) and \( F_e \) are, respectively, the projections of \( R_p \) on the lines perpendicular to the internal and external trochlear cheeks.

Biomechanical Data

This section presents the results of research done by other authors. These results will be useful later as indications for the present model.

1. Huberti et al.\(^9\) showed that the intensity of \( F_p \) and \( F_q \) are not always equal. In fact,
with the values \((k, \theta)\): \((1,0^\circ)\) \((1.27,30^\circ)\) \((0.91,60^\circ)\) \((0.70,90^\circ)\).

2. The patellar-femoral contact pressure is highest between 60° and 90° of flexion (see Huberti and Hayes).

3. Chondromalacial lesions occur most frequently in regions corresponding to contact areas at between 40° and 80° of flexion (see Seedhom et al.).

4. Fujikawa et al. studied the pattern of contact and the congruence of the patellar-femoral joint. Their work shows that 1) the lateral side has the largest contact area and 2) the patellar facet angle and the trochlear femoral angle show different values at contact sites for different angles of flexion. Therefore, \(S_i\) and \(S_e\) depend on \(\theta\).

5. Patellar dislocation occurs at the beginning of knee flexion.

**Practical Constraints**

CT data are acquired with the subject’s knee in full extension. This is significant because the angle of flexion is near 0° and all \(S_i\), \(S_e\), \(\alpha_i\), \(\alpha_e\), \(F_i\), \(F_e\) vary according to \(\theta\). Although Garg and Walker have made a three-dimensional (3-D) model of knee motion using cadavers, a 3-D dynamic study has not been made because it entails increasing the time of patient examinations, thus increasing X-ray exposure, cost, time, and services from personnel and equipment. Although in the future a 3-D dynamic model will have to be developed for the patellar-femoral joint, for the present we can use the data already available. Thus we consider the biomechanical configuration as constant. This hypothesis is less restrictive than it appears to be, because patellar dislocations occur at the beginning of flexion (\(\theta < 10^\circ\)), and the data were acquired with the knee in full extension (\(\theta \approx 0^\circ\)).

Let us define a reference coordinate system in which the \(x\) axis is the lateral-medial axis and the \(y\) axis is the sagittal axis. The origin \(G_i\) is the intersection of the two trochlear cheeks in the plane \(P\). \(\alpha_i\) is the angle between the \(x\) axis and the trace of the internal side of the trochlea in plane \(P\), \(\alpha_e\) is the angle between the \(x\) axis and the trace of the external side of the trochlear in plane \(P\). The classical notation considers \(\alpha_i\) and \(\alpha_e\) as the angles between the line representing the coronal plane tangent to both femoral condyles and the lines representing the external or internal sides of the trochlea, respectively (see Fig. 4).

**Biomechanical Hypotheses**

Let \(\phi\) be the angle between \(x\) and \(R\), and \(\phi_i\) and \(\phi_e\) the angles between \(x\) and, respectively, \(F_i\) and \(F_e\) (Fig. 5). The quadriceps by itself dynamically stabilizes the patella if the force is situated in the security triangle delimited by the two perpendiculars to the two trochlear cheeks. Depending on which CT images are used, it is necessary to distinguish between right and left knees. This dynamic stabilization is ideal if the \(F_i\) and \(F_e\) forces (which are the projections of \(R\) on the lines perpendicular to the trochlear cheeks) are proportional to the external \(S_e\) and internal \(S_i\) surfaces of these trochlear cheeks. Therefore, ideally, the two facets of a stable and correctly balanced patella exert the same pressure on the two corresponding cheeks of the trochlea (see Fig. 5). It is not possible to measure exactly the values \(S_i\) and \(S_e\) using CT data, which is why the assumption is made that the ratio \(\frac{S_i}{S_e}\) is equal to \(\frac{L_i}{L_e}\). \(L_i\) and \(L_e\) represent \(S_i\) and \(S_e\) in the plane \(P\) (for experimental results, see Maquet and Townsend et al.). The values \(L_i\) and \(L_e\) are easily identified on a determined CT slice.

We can summarize the two hypotheses as follows. Hypothesis 1 (hypothesis of minimal constraint): \(R_p\) is in the security triangle; this implies:

\[
\begin{align*}
\phi_i & \geq \phi \geq \phi_e \quad \text{(right)} \\
\phi_e & \leq \phi \leq \phi_i \quad \text{(left)}
\end{align*}
\]

Hypothesis 2 (hypothesis of ideal constraint): \(R_p\) makes an angle \(\phi_{ideal}\) with the \(x\) axis such that

\[
\frac{F_i}{L_i} = \frac{F_e}{L_e}
\]
Surgical Constraint

Lateral patellar dislocation or lateral patellar-femoral arthrosis can be corrected by a patellar tendon transposition on the inner side of the tibial tuberosity. To determine the amplitude of this transposition, a mathematical model of this problem is proposed using the two hypotheses given above. Nevertheless, we offer additional hypotheses that express surgical constraint and experimental knowledge. If one considers that one of the two points that gives the direction of \( F \), is the tibial insertion of the tendon on the tibial tuberosity, transposing this point implies modifying \( F \).

We choose 1) to impose no proximal or distal displacement of the patellar tendon, so that the \( z \) coordinate of \( F \) is constant (When the patella is too high, the surgeon lowers it; it is a distal transposition of the ATT.) and 2) to have no Maquet or anti-Maquet effect; this implies that the \( y \) coordinate of \( F \) is constant (when patellar-femoral arthrosis is present, the surgeon can place the ATT anteriorly). The study is done in the case of a simple transposition along the \( x \) axis with the knee in full extension.

MATHEMATICAL MODEL

Notation

The reference coordinate system is defined by: the \( x \) axis (medial/lateral), the \( y \) axis (anterior/posterior), the \( z \) axis (proximal/distal axis), and the origin \( G \). \( F \) is \(( F_x, F_y, F_z )\). \( F_i \) is \(( F_{i,x}, F_{i,y}, F_{i,z} )\). \( F_i^* \) \(( F_{i,x}^*, F_{i,y}^*, F_{i,z}^* )\) is the force created by the patellar tendon after transposition (minimal case: \( F_i^* = F_i^{ideal} \), ideal case: \( F_i^* = F_i^{minimal} \)).

\( R \) is \(( R_x, R_y, R_z )\); \( R^* \) \(( R_x^*, R_y^*, R_z^* )\) is the result of \( F \) and \( F_i \). \( R^* \) \(( R_x^*, R_y^*, R_z^* )\) is the projection of \( R \) in plane \( P \).

\( F_i \) \(( F_{i,x}, F_{i,y}, 0 )\) and \( F_z \) \(( F_{z,x}, F_{z,y}, 0 )\) are the orthogonal \( R^* \) projection on the internal and external trochlear cheeks in \( P \). \( R^*_p \) \(( R^*_p,x, R^*_p,y, 0 )\) is the projection of \( R^* \) in plane \( P \). \( R^*_{p,ideal} \) is the projection of \( R^*_{minimal} \) is the projection of \( R^*_{minimal} \).

Direction of \( R^* = R^*_{minimal} \) According to Hypothesis 1

Wherever the \( R^* \) force is applied, we consider the angle formed by the direction of the force relative to the \( x \) axis. \( R^* \) must be in the security triangle. In the simplest case, this angle corresponds to the angle between the \( x \) axis and \( F \), and is denoted \( \phi_{minimal} \) (see Fig. 6). For the right knee, \( \phi_{minimal} = -\pi/2 - \alpha_x \). For the left knee, \( \phi_{minimal} = -\pi/2 + \alpha_x \).

Direction of \( R^* = R^*_{ideal} \) According to Hypothesis 2

For the right knee, because \( \frac{L_i}{L_z} = \frac{S_i}{S_z} \) one has a direct relationship: \( \phi_{ideal} = \alpha_x, \alpha_y, L_x, \) and \( L_y \).

\[
\tan(\phi_{ideal}) = \frac{L_i \cdot \cos(\alpha_y) + L_z \cdot \cos(\alpha_x)}{L_i \cdot \sin(\alpha_x) - L_z \cdot \sin(\alpha_y)}
\]  

For the right knee \( \phi = 1 \); for the left knee \( \phi = -1 \).

Mathematical Relation Involving Force Components

Transposition according to the surgical constraint is possible only along the \( x \) axis. Let \( \phi \) be \( \phi_{minimal} \) or \( \phi_{ideal} \) and we have the relation:

\[
F_{i,x}^* + F_{q,x} = R_x^* = R^* \cdot \cos(\phi^*)
\]

\[
F_{i,y}^* + F_{q,y} = R_y^* = R^* \cdot \sin(\phi^*)
\]

which gives:

\[
F_{i,y}^* = -F_{q,y} + \cot g(\phi^*) \cdot (F_{i,y} + F_{q,y})
\]
ACQUIRING NUMERICAL DATA USING CT IMAGES

Data Available with Computed Tomography

The data obtainable are a set of CT slices acquired with the patient lying on the scanner table with the knee in full extension (see Figs. 11, 12). In the reference coordinate system of the scanner, we can extract from these slices (see Fig. 7) 1) the coordinates of the tibial insertion of the patellar tendon. (mean of three slices; see Figs. 12, 13); this point is denoted \( T = (T_x, T_y, T_z) \); 2) information about the contact areas' between trochlea and patella, i.e., the length of the external and internal trochlear cheeks \( L_e, L_i \), and the angles these cheeks make with the \( x \) axis, \( \alpha_e, \alpha_i \). The coordinates of the intersection point of these two cheeks are denoted \( G = (G_x, G_y, G_z) \); (see Fig. 15); and 3) a set of \( N_s \) slices \((N_s \text{ ranges from 8 to 11})\) containing information about the quadriceps. It is possible to determine the surface of the quadriceps and the gravity center \( Q \) of the quadriceps on each surface (see Fig. 16).

The reference coordinate system of the scanner, denoted \( R_s \), is the same as that defined above under Notation, with only a translation between the origin of the scanner and the point \( G \). This small difference does not affect the calculus because vectorial coordinates are used.

Features of \( F_t \)

The direction of \( F_t \) is defined by two points. One is \( T_s = (T_{x_s}, T_{y_s}, T_{z_s}) \). For the second, we successively opted 1) for the bottom point of the trochlear groove, although this point is not adequately representative from a biomechanical standpoint; 2) for the patella center of gravity, although Merchant et al.\(^{19}\) showed that the patella moves significantly depending on the contraction or relaxation of the quadriceps and that the measure is not repeatable; and 3) finally for the fictive point defined as the intersection of the two medians of the trochlear cheeks (see Figs. 8, 14). This point is denoted \( S_i \) (See Fig. 8)

The coordinates of \( S_i \) are \( (S_{x_i}, S_{y_i}, S_{z_i}) \) with:

\[
S_{x_i} = G_{x_i} + \frac{\varepsilon \cdot L_e \cdot \sin(\alpha_e) - \varepsilon \cdot L_i \cdot \sin(\alpha_i)}{2 \cdot (\cos(\alpha_e) \cdot \sin(\alpha_e) + \cos(\alpha_i) \cdot \sin(\alpha_i))} \\
S_{y_i} = G_{y_i} + \frac{\varepsilon \cdot L_e \cdot \cos(\alpha_e) - \varepsilon \cdot L_i \cdot \cos(\alpha_i)}{2 \cdot (\cos(\alpha_e) \cdot \sin(\alpha_e) + \cos(\alpha_i) \cdot \sin(\alpha_i))} \\
S_{z_i} = G_{z_i}
\]

\( \varepsilon = 1 \) for the right knee, and -1 for the left knee. Finally, \( F_t \) is colinear to the vector \( S_i \cdot T_s \). It exists \( \lambda \in R^+ \) such that

\[
F_t = \lambda \cdot S_i \cdot T_s \tag{7}
\]

Features of \( F_q \)

We assume that the direction of \( F_q \) is given by the point \( S_i \) and the quadriceps center of gravity (QGC) \( Q_0 = (Q_{x_0}, Q_{y_0}, Q_{z_0}) \). Let us call \( D \) the domain of the quadriceps; we have:

\[
Q_{var} = \frac{\int_D var \cdot dx \cdot dy \cdot dz}{\int_D dx \cdot dy \cdot dz} \tag{8}
\]

(var is either \( x \), \( y \), or \( z \)). To make an evaluation of the center of the QGC, we take into account each scanner slice where the quadriceps appears. On each slice referenced by \( i = 1 \ldots n \), the quadriceps is detected on an area \( A_i \) associated with a height called \( Z \), and a thickness \( dh \). We call \( Q_{x_i} = (Q_{x_{i_0}}, Q_{x_{i_1}}, Q_{x_{i_2}}) \) the center of gravity of the part of the quadriceps appearing in the slice \( i \). Let \( S_{slice_i} = S_{slice_i}(x, y) \) the ap-
Fig. 7. Description of the CT slices.

Fig. 8. Determination of the $S_i$ point.
proximated coordinates of $Q_z$ in $R_o$ are given by:

$$Q_{z,w} = \frac{\sum w_i Q_{z,w} \cdot S_{slice_i}}{\sum w_i S_{slice_i}}$$  \hspace{1cm} (9)

(var is either $x$, $y$, or $z$). Finally $F_q$ is colinear to the vector $S_{Q_q}$. It exists $\lambda' \in R^+$ such that

$$F_q = \lambda' \cdot S_{Q_q}$$  \hspace{1cm} (10)

Let $f^*$ be

$$f^* = \frac{\|S_{T_s}^*\|}{\|S_{Q_q}\|} \text{ then } \frac{\gamma'}{\lambda} = \frac{f^*}{k(\theta)}$$  \hspace{1cm} (11)

with $k(\theta) = 1$, because $\theta$ is near $0^\circ$.

**VALUE OF THE PATELLAE TENDON TRANSPOSITION**

**Determination of the New Tibial Insertion**

Let $T_{s,i}^*$ be the new point of insertion with the coordinates $(T_{s,i}, T_{s,i}, T_{s,i})$. By using equations 6, 7, 10, and 11, the final formula becomes:

$$T_{s,i}^* - S_{s,i} = -f^* (Q_{s,i} - S_{s,i}) + \cot g(\phi^*) (T_{s,i} - S_{s,i} + f^* (Q_{s,i} - S_{s,i}))$$  \hspace{1cm} (12)

The difference $\Delta x$ such that:

$$\Delta x = T_{s,i}^* - T_{s,i} = F_{s,i} - F_{s,i}$$  \hspace{1cm} (13)

indicates the amplitude of possible transposition on the inner side of the tibial tuberosity.

**Uncertainties**

To deal with uncertainties on scanner data measurements, we differentiated $\Delta x$ relative to $L_s$, $L_s$, $\alpha_s$, $\alpha_s$, $S_s$, $S_s$, $Q_s$, $Q_s$. The error is estimated to about 1 mm for $L_s$, $L_s$, $S_s$, $S_s$; 3 mm for $Q_s$, $Q_s$; and about 1° for $\alpha_s$, $\alpha_s$.

To achieve $Q_{s,i}^*$, $Q_{s,i}^*$, $Q_{s,i}^*$, we use a segmentation interface, which allows each area $A_i$ to be determined for each CT slice $i$. The quadriceps is manually segmented, and some errors can occur. A repetitive test for the segmentation gave good results, because the coordinates of $Q_s$ vary by less than 3 mm.

Another origin of errors is the state of the quadriceps during the scan; i.e., is the quadriceps relaxed or contracted? The data were acquired from five patients with quadriceps contracted ($Q_{p,c}$) and then relaxed ($Q_{p,r}$). The result is that the distance between $Q_{p,c}$ and $Q_{p,r}$ is less than 3 mm, so the state of the quadriceps has a negligible influence in determining $Q_s$ (see Fig. 10).

**RESULTS**

**Results from a 3D Representation**

Figure 9 explains Figure 10, which shows the 3-D position of the data used. Figure 9 was obtained with the AVS software. In Figure 10, data for both contracted and relaxed quadriceps are presented. We have represented the line (MS line) that fits the set of intermediate gravity centers to the mean square criterion.

This representation shows that 1) a line exists that joins the intermediate gravity centers with very good approximation, 2) the center of gravity $Q_s$ is really near the middle of the MS line (the distance between $Q_s$ and MS is less than 3 mm), and 3) the direction of MS passes beneath the trochlear cheeks. In fact, the quadriceps is made of four heads [vastus intermedius (VI), rectus femoris (RF), vastus medialis

![Fig. 9. Line passing by the center of gravity of each slice.](image-url)
Every head has fibers with individual directions. The final direction is a combination of the four directions. We assume that this final direction of $F_q$ is given by the line passing by $S$, and $Q_s$; this seems to be the most useful compromise. Also, the direction of $F$, (given by $S$, and $T$) and $P$ effectively shows the possibility of lateral subluxation.

**Numerical Results**

Table 1 contains 1) the patient number in column Pat; 2) the concerned side (L for left, R for right) in column S; 3) the value of the Q angle (in degrees) in column Q angle; 4) the value of the ATTG (in millimeters) in column ATTG; 5) in the ST column, Y means that $R_s$ is in the security triangle, and N means that $R_s$ is not in the security triangle (in the latter case, there is a possibility of lateral subluxation); 6) the value (in millimeters) of the transposition in the case of hypothesis 1 in column $\Delta x_{\text{min}}$; 7) the value (in millimeters) of the transposition in the case of hypothesis 2 in column $\Delta x_{\text{ideal}}$; 8) the error (in millimeters) made on the calculus in column Error; 9) the
value (in degrees) of the Q angle calculated for $\phi_{\text{minimal}}$ in column $Q_{\text{minimal}}$; 10) the value (in degrees) of the Q angle calculated for $\phi_{\text{ideal}}$ in column $Q_{\text{ideal}}$; 11) the value (in millimeters) of the ATTG calculated for $\phi_{\text{minimal}}$ in column $ATTG_{\text{minimal}}$; and 12) the value (in millimeters) of the ATTG calculated for $\phi_{\text{ideal}}$ in column $ATTG_{\text{ideal}}$.

A classical norm for the ATTG is 10–15 millimeters and 15° for the Q angle. In most the cases (patients 13 and 15 are exceptions), the projection of the $R_p$ force is out of the security triangle. These results are in accordance with the pathological phenomenon. The computed inner transposition (positive for the right knee and negative for the left knee) is useful in most cases, as shown in column $\Delta x_{\text{max}}$, and in most cases is too high, as shown in column $\Delta x_{\text{ideal}}$.

In fact, in the minimal case, the inner transposition tends to normalize both the Q angle and the ATTG values. In the ideal case, the Q angle is near 0° and the ATTG is near 0 mm. The results of the ideal case correspond to the state reached by a normal patellar-femoral joint near 90° of flexion.

It seems that the ideal case introduces a very strong constraint. The hypothesis of equipressure is very important when the contact pressure is maximal (between 60° and 90° of flexion). At this stage, the configuration is completely different from that when the knee is in full extension, because there is an abduction of the knee of about 20°. Experiments have been inconclusive regarding ideal constraints.

Movement of the patellar-femoral joint will have to be dealt with in the future using a 3D model. The hypothesis of equipressure (ideal constraint) should be studied between 60° and 90° of flexion where chondromalacia occurs most frequently. The
minimal constraint is well adapted in our configuration (knee near 0° of flexion) because it provides a good model for correction of subluxation, which occurs at the beginning of flexion.

Future Work

Determination of the center of gravity of the quadriceps is a very delicate part of our method. Even with software adapted to segment the quadriceps on every CT slice and to calculate the area, the segmentation time is too great. Moreover, reducing manipulation by radiology personnel reduces costs. To simplify the procedure, it is possible to acquire only three CT slices: 1) one passing through the middle position of the quadriceps, 2) one passing through the mean attachment point of the tibial tendon on the tibial tuberosity, and 3) one passing through the contact area between the patella and the femur.

With these three slices, it is possible to extract all the data needed. Tests have been done to evaluate the error generated by this simplification. The only difference concerns QGC determination. As was explained above under Results From a 3D Representation, Qₕ is quite near the MS line. Taking the middle of the segment QₚₕQₑₕ as Qₙ generates an error smaller than 2 mm on Δx. For this reason, the QGC of the slice passing through the middle of the quadriceps gives a good estimation of the global QGC at present determined by Nₙ intermediate slices.

It would be interesting and complementary to work on data acquired with the knee at 90° of flexion using the same model, especially when the equipressure constraint is applied. In using the simplification described above, only six slices for each patient are needed, which is a good compromise regarding X-ray dose, cost, and time. The main interest will lie in comparing Δxₐ₁₁ with the knee at 90° of flexion and Δxₜₐ₅ with the knee at 0° of flexion.

CONCLUSIONS

A new model for the patellar-femoral joint has been developed and can be used as a starting point for other accurate models. By using CT data, it is possible to compute the ATTT with respect to two criteria, which depend on a biomechanical model and biomechanical hypotheses. According to the criteria of minimal constraint (correction of subluxation), the computed value of the transposition fits well with the concept of the ATTT and the common values (5–15 mm) applied during interventions. The criteria of ideal constraint (equipressure of the two trochlear cheeks) gives excessive values for the inner transposition (15–25 mm). In fact, the CT data acquired with the knee fully extended should not be used with this criterion. In the future, it will be necessary to add CT data with the knee at 90° of flexion and to observe whether the two criteria (minimal at 0° and ideal at 90°) are redundant. In any case, these preliminary results are a link between a diagnosis of patellar dislocation and surgical interventions that entail an inner transposition of the ATT. Theoretical research and clinical work have to be continued for model and laboratory validations.

---

Table 1. Numerical Results Obtained from Clinical Data

<table>
<thead>
<tr>
<th>Pat.</th>
<th>S.</th>
<th>Q angle</th>
<th>ATTT</th>
<th>ST</th>
<th>Δxₐ₁₁</th>
<th>Δxₛ₁₁</th>
<th>error</th>
<th>Q min</th>
<th>Q ideal</th>
<th>ATTT min</th>
<th>ATTT ideal</th>
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<tr>
<td>1</td>
<td>L</td>
<td>26.3</td>
<td>24.0</td>
<td>N</td>
<td>-11.0</td>
<td>-19.6</td>
<td>4.2</td>
<td>15.1</td>
<td>5.6</td>
<td>13.0</td>
<td>4.4</td>
</tr>
<tr>
<td>2</td>
<td>R</td>
<td>20.6</td>
<td>18.0</td>
<td>N</td>
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<td>17.4</td>
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<td>3.3</td>
<td>4.5</td>
<td>-5.9</td>
</tr>
<tr>
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<td>L</td>
<td>30.0</td>
<td>23.0</td>
<td>N</td>
<td>-8.2</td>
<td>-17.3</td>
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The possibility of mathematical determination of patellar ligament tibial insertion has a twofold interest. It should eliminate anterior peripatellar knee pain by accurately identifying the excessive lateral pressure syndrome well described clinically by P. Ficat, and it should be useful to surgeons in avoiding excessive transpositions.

ACKNOWLEDGMENTS

In 1991, Miss Gaborit died in an avalanche. We hope the contribution her work and personal generosity made to this project is recognized not only here but in the hereafter.

REFERENCES